FORMATION FLYING WITH DECENTRALIZED CONTROL
IN LIBRATION POINT ORBITS

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ABSTRACT - A decentralized control framework is investigated for applicability of formation flying control in libration orbits. The decentralized approach, being non-hierarchical, processes only direct measurement data, in parallel with the other spacecraft. Control is accomplished via linearization about a reference libration orbit with standard control using a Linear Quadratic Regulator (LQR) or the GSFC control algorithm. Both are linearized about the current state estimate as with the extended Kalman filter. Based on this preliminary work, the decentralized approach appears to be feasible for upcoming libration missions using distributed spacecraft.

1 – INTRODUCTION AND BACKGROUND

Libration orbit missions in the near future will use multiple spacecraft in a distributed approach to perform interferometry and optical measurements not achievable by single spacecraft. We investigate formation flying concepts in a Sun-Earth libration orbit dynamical environment for these future missions. The Guidance, Navigation, and Control Center (GNCC) at the Goddard Space Flight Center (GSFC) is developing and implementing enhanced autonomous formation flying systems which improve operations while minimizing impacts to onboard software and hardware requirements, software development, and integration ([Folt98], [Carp99], [Carp00]). In future missions, distributed systems are expected to require a communications cross-link for sharing housekeeping telemetry, navigation data, and measurements. In anticipation of such missions, the GSFC GNCC is investigating an option for closed-loop autonomous navigation and maneuver control of satellite formations that are based on the decentralized framework developed in [Spey79]. We begin by providing a brief description of libration orbit simulation, then describe GSFC decentralized control methods useful for support of these missions. We demonstrate new developments such as GSFC's formation control algorithm and traditional linear quadratic control methods which are incorporated into the decentralized approach.

Problems of single spacecraft control of circular restricted three-body (CRTB) motion have been previously investigated using state-space equations to characterize the linearized equations of motion ([Hoff93], [Farq70], [Wie98]). State-space analysis methods in control theory provide a useful framework for defining goals and the optimal control of satellites designed to fly about a reference orbit. We advance formation flying control using state-space by incorporating two control algorithms into a decentralized architecture. The mathematical foundation for the explanation of CRTB motion is briefly addressed, but only to the degree required to understand the results of the simulations presented. We consider the effects of third body and solar radiation perturbations into the state space simulations and calculate command responses, including disturbance rejection, on the formation of two spacecraft.

The decentralized approach is non-hierarchical, eliminating the need for coordination by a central supervisor and permits graceful degradation of system performance in the presence of detected

failures. Each spacecraft (spacecraft) in the decentralized network processes only its own measurement data, in parallel with the other spacecraft. Although the total computational burden over the entire network is greater than it would be for a single, centralized controller, fewer computations are required locally at any individual spacecraft. Requirements for data transmission between spacecraft are minimized, at the cost of locally maintaining an additional data vector. This data vector retains a memory of all past measurement history from all of the spacecraft compressed into a single vector with the dimension of the state.

2 – LIBRATION ORBITS

A wealth of references can be readily found on methods to derive the principle equation for the dynamic behavior of a single spacecraft in orbit around the Sun-Earth co-linear libration point, $L1$. These methods using Hamiltonian, Jacobian or Newtonian approaches start with the same initial assumptions: an infinitely small mass is placed in the gravitational field of two massive bodies or primary bodies. These methods are generally referred to as the restricted three-body (RTB) problem. Furthermore, if it is assumed that the primary bodies follows a circular orbit around their barycenter, then this is called the circular restricted three-body (CRTB) problem. An appropriate coordinate system needs to be selected.

2.1 – A State Space Model

As with the current literature about this subject ([Hoff93] [Wie98]), we use three-dimensional Cartesian coordinates denoted by capital letters to describe a system with an origin at the barycenter of the primary bodies and small letters will be used for an origin at a libration point. Figure 1 illustrates the coordinate system and sample orbit that will be used here. Here m is an infinitely small mass in the gravitational field of the primary bodies M_1 and M_2.
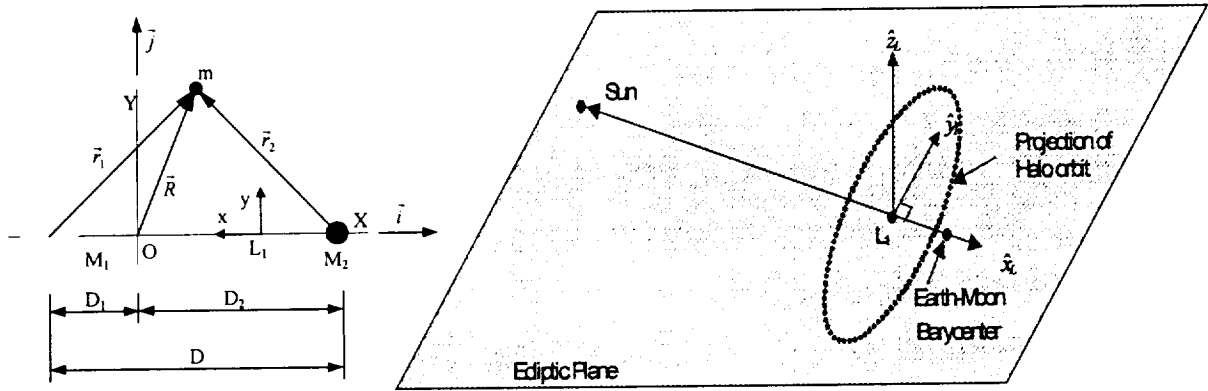

Fig. 1: Coordinate system and sample libration orbit

The linearized equation of motion for m close to the libration point is

$$\ddot{x} - 2n\dot{y} = U_{XX}x , \quad \ddot{y} + 2n\dot{x} = U_{YY}y , \text{ and } \quad \ddot{z} = U_{ZZ}z \tag{2.1}$$

where $U_{xx} = \dfrac{\partial^2 U}{\partial X^2}$, $U_{YY} = \dfrac{\partial^2 U}{\partial Y^2}$ and $U_{ZZ} = \dfrac{\partial^2 U}{\partial Z^2}$. U_{XX}, U_{YY} and U_{ZZ} are calculated at the respective libration point to get the respective equation of motion and are constants. As a result of the linearization, x and y are coupled whereas z is now completely independent and is a simple harmonic. These equations can also be written in state-space form as below where x' represents the

$$\dot{\mathbf{x}}^{\prime} = A^{\prime}\mathbf{x}^{\prime} \quad \text{where} \quad \mathbf{x}^{\prime} = \begin{bmatrix} x^{\prime} \\ y^{\prime} \\ z^{\prime} \\ \dot{x}^{\prime} \\ \dot{y}^{\prime} \\ \dot{z}^{\prime} \end{bmatrix}, \quad A^{\prime} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ U_{xx} & 0 & 0 & 0 & 2n & 0 \\ 0 & U_{YY} & 0 & -2n & 0 & 0 \\ 0 & 0 & U_{ZZ} & 0 & 0 & 0 \end{bmatrix} \tag{2.2}$$

state of the jth spacecraft in the formation. This is the form that will be used in the following sections.

2.3 – Reference orbit

In order to compare our simulations using the state space matrix to a reference and to have a reference orbit to control a spacecraft to, it is essential to find a proper reference model. The model used generally in the literature has an oscillating form. The amplitudes and phases of this reference model depend on the initial conditions of the simulation.

$$
\begin{aligned}
x &= -A_x \sin(\omega_{xy}t + \varphi_{xy}) & \dot{x} &= -A_x\omega_{xy}\cos(\omega_{xy}t + \varphi_{xy}) \\
y &= -A_y \cos(\omega_{xy}t + \varphi_{xy}) & \dot{y} &= A_y\omega_{xy}\sin(\omega_{xy}t + \varphi_{xy}) \\
z &= A_z \sin(\omega_z t + \varphi_z) & \dot{z} &= A_z\omega_z\cos(\omega_z t + \varphi_z)
\end{aligned}
\tag{2.3}
$$

2.4 – Simulation: The Circular Restricted Three-Body Problem for Earth-Sun L1

To visualize the results above in graphs, a simulation is implemented with MATLAB/SIMULINK that will calculate the motion and plot the results. To achieve more accurate results in a CRTB, the Earth system will consist of the mass of Earth plus the mass of moon. This assumption is generally accepted by [Sze67], [Hoff93] and [Wie98]. Using the equations above it can the be computed that:

$M_1=1.9891\times10^{30}$ kg, $M_2 =6.0477\times10^{24}$ kg (Mass of Sun and Moon)

$U_{XX}=3.6262\times10^{-13}$ m/s^2, $U_{YY}=-1.2185\times10^{-13}$ m/s^2, $U_{ZZ}=-1.6149\times10^{-13}$ m/s^2

$\omega_{xy} = 4.1603\times10^{-7}$ rad/s $\approx T_{xy} = 174.8$ days and $\omega_z= 4.01855\times10^{-7}$ rad/s $\approx T_z = 180$ days

With these values a libration orbit is propagated over five years. Figure 2 shows the result of this simulation. The simulation error compared to the reference model if expressed as a percentage of the absolute distance in each direction, would not exceed 0.067%. The solar radiation pressure and the effects from the Moon and Jupiter were not included in the formation control aspect of this paper, but should be considered in future analysis.
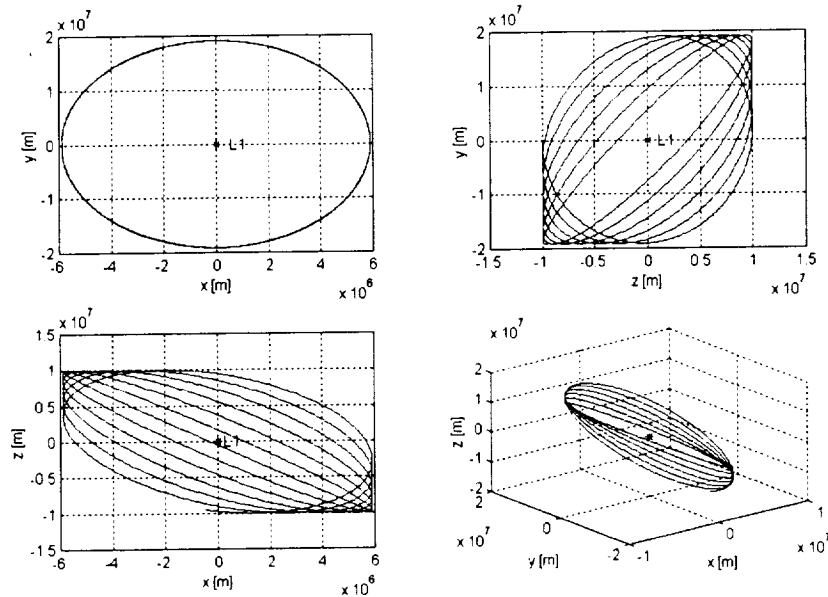

Fig. 1: Libration orbit around Sun-Earth L1 without perturbations

3 – CONTROL ALGORITHMS

The analysis presented here is based on generalizing earlier work [Hoff93] [Wie98] to the task of distributed spacecraft at the Sun-Earth, rather than Earth-Moon, collinear equilibrium point. In [Carp00], the decentralized LQG framework developed [Spey79] is applied to an idealized (linear,

time-invariant) version of a low Earth orbit satellite formation control problem. In this paper, the approach taken in [Carp00] is applied to a centralized design for a formation of satellites at the Sun-Earth $L1$ point, which is based on the work of [Hoff93] and [Wie98]. Also, the Folta-Quinn (FQ) orbit control algorithm of [Folt98] is substituted for the LQR control used by [Carp00], as was done in [Carp99]. This is accomplished by treating the framework of [Spey79] as a decentralized estimator only; globally optimal states, rather than globally optimal controls, are then reconstructed at each spacecraft for use by the FQ algorithm.

3.1 – Centralized LQR Design

To maintain a spacecraft in orbit about $L1$ a control can be introduced that provides the propulsion system the direction and magnitude of the thrust in order for it to stay as close as possible to a reference trajectory. One such control used by spacecraft is the Linear Quadratic Regulator (LQR). The LQR is a control algorithm that uses state feedback to keep a system like a spacecraft to an equilibrium condition and then holds it there despite disturbances. State feedback indicates that the control command is directly proportional to the state. In the case of a spacecraft, the control command would be the thrust and the state would be the position and velocity of the spacecraft relative to the libration point. The reference orbit would define the equilibrium condition [Wie98].

The state dynamics of the controlled spacecraft are described by

$$\dot{\mathbf{x}}^j = A^j \mathbf{x}^j + B^j \mathbf{u}^j + \mathbf{a}_{solpres} + \mathbf{a}_{FB} \tag{3.1}$$

where the matrix B^j maps the control input from the control space to the state space. To keep the spacecraft as close to the reference orbit, $\mathbf{x}^R$, as possible, and to minimize fuel usage, the performance index

$$J = \frac{1}{2} \int_0^\infty \{(\mathbf{x} - \mathbf{x}^R)^T Q (\mathbf{x} - \mathbf{x}^R) + \mathbf{u}^T R \mathbf{u}\} dt \tag{3.2}$$

is minimized. To control a formation of satellites, two approaches are possible. Either each vehicle can be individually controlled to its own reference trajectory about $L1$, or the spacecraft relative positions can be controlled. In the former case, the control problem can be decoupled such that each vehicle's optimal control is independent of the other spacecraft states and controls. Then, $\mathbf{x}$ and $\mathbf{u}$ above represent a single spacecraft state and control $\mathbf{x}^j$ and $\mathbf{u}^j$. Otherwise, $\mathbf{x}$ and $\mathbf{u}$ above represent all the spacecraft states and controls, $\mathbf{x} = [\mathbf{x}^1; \mathbf{x}^2; ...; \mathbf{x}^K]$, $\mathbf{u} = [\mathbf{u}^1; \mathbf{u}^2; ...; \mathbf{u}^K]$. In either case, the solution to this problem is

$$\mathbf{u}^j = -\left[R^j\right]^{-1}\left[B^j\right]^T S \mathbf{x} \tag{3.3}$$

where S is the steady-state solution of the algebraic Riccati equation for time invariant problems

$$S(B R^{-1} B^T)S - SA - A^T S - Q = 0$$

The solution to this equation is calculated using the MATLAB/Control Systems Toolbox function *lqr*. The *lqr* function needs the A, B, Q, and R matrices as its input. The matrix A has been defined previously for a single spacecraft. For multiple spacecraft, A is assumed to be equivalent for each spacecraft as the separation distance between spacecraft for our analysis is much smaller that the distance from either the Earth or Sun. If all the spacecraft are to be simultaneously controlled, A is defined to include a copy of the single spacecraft A^j matrix for each vehicle in the formation along its diagonal.

The B matrix maps the control input from the control space to the state space, e.g. B determines which states are influenced directly by the control. For the centralized controller design model it will be assumed that the spacecraft thrusts continuously and that the thrust level can vary continuously in time. Also, the spacecraft's control system can apply the thrust directly in each component of the coordinate system [Wie98]. Therefore, the B matrix for an individual spacecraft is given by

$$B' = \begin{bmatrix} O_{3\times3} \\ I_3 \end{bmatrix} \qquad (3.4)$$

If all the spacecraft are to be simultaneously controlled, B is defined to include a copy of the single vehicle's B' matrix for each vehicle in the formation along its diagonal.

The Q and R matrix are performance weights for the LQR. Q is the weighting matrix for the state error, which determines how accurate the reference is followed, and R is the weighting matrix for the control effort, which determines the damping of the system. In both cases trial and error and a certain amount of experience determine the selection of Q and R. There are however some guidelines proposed by [Wie98] on how to proceed. Let

$$Q^J = \begin{bmatrix} \dfrac{1}{p}I_3 & O_{3\times3} \\ O_{3\times3} & \dfrac{1}{q}I_3 \end{bmatrix}, R^J = \dfrac{1}{r}I_3 \qquad (3.5)$$

The value of r determines how well the controlled system is damped. If p and q are selected, then r can be used to tune the controller until it produces reasonable results. In the present case, $p = 100$ m^2, corresponding to a position tolerance of 10 m, and $q = 0.0001$ (m/s)^2 corresponding to a velocity tolerance of 0.01 m/s. If all the spacecraft are to be simultaneously controlled, Q and R are defined to include copies of the single vehicle's Q^J and R^J matrices for each vehicle in the formation along their diagonals.

3.2 – Disturbance Accommodation Model

The spacecraft thrusts continuously due to the perturbing forces and because the reference model does not correspond with the numeric solution exactly. To save fuel it is however desirable to decrease thrusting time. [Hoff93] and [Wie98] have similar methods to limit fuel consumption by introducing a disturbance model. This technique was successfully used to improve the attitude control design for the International Space Station.

The reason that the LQR design cannot keep the thrust at a lower level is that the A matrix does not include the disturbances from the perturbations and the fact that the simulation does not equal the reference model exactly. The real solution has additional nonlinear terms that cause the controller to signal a higher thrust. A disturbance model can reduce the effects of these terms and reduce the thrust. The disturbance accommodation model allows the states to have non-zero variations from the reference in response to the perturbations without inducing additional control effort. In essence, the additional states introduced by the disturbance model are in resonance with modes of the control effort caused by the disturbances. As the disturbance states are stimulated, they absorb control effort, so that in steady-state, the control effort goes to zero.

The state matrix for the disturbance model can be written as

$$\dot{\delta}^J = A^J_{dist}\delta^J \qquad (3.6)$$

with
$$\delta^J = \begin{bmatrix} \alpha^J \\ \beta^J \\ \tau^J \end{bmatrix}, \quad A^J_{dist} = \begin{bmatrix} O_{n\times n} & I_n & O_{n\times m} \\ diag(\omega^J) & O_{n\times n} & O_{n\times m} \\ O_{m\times n} & O_{m\times n} & O_{m\times m} \end{bmatrix}$$

where $\alpha^J = [\alpha_1;\ldots;\alpha_n]$ and $\beta^J = [\beta_1;\ldots;\beta_n]$ represent n periodic disturbances with frequencies given by $\omega^J = [\omega_1;\ldots;\omega_n]$, and $\tau^J = [\tau_1;\ldots;\tau_m]$ represent m constant disturbances. To completely accommodate all disturbances the model would have to include many periodic disturbances and integral states. This however would make the model very complex and computational expensive. Therefore, only the most severe disturbances are included in this model.

The task at hand is to find a suitable set of frequencies. A method that is described in [Wie98] will be used to calculate these unknowns. The periodic disturbances are determined by calculating the power spectral density of the optimal control for the motion of a spacecraft around Sun-Earth $L1$. An optimal control would keep the spacecraft right on the reference orbit using the least amount of fuel. [Hoff93] substitutes $\mathbf{x}'$ and $\dot{\mathbf{x}}'$ with the reference states to calculating the optimal $\mathbf{u}'$, e.g. solving

$$\dot{\mathbf{x}}^{Rj} = A^j \mathbf{x}^{Rj} + B^j \mathbf{u}^j + \mathbf{a}_{solpres} + \mathbf{a}_{FB} \tag{3.7}$$

and for $B^j \mathbf{u}^j$

$$B^j \mathbf{u}^j = \dot{\mathbf{x}}^{Rj} - A^j \mathbf{x}^{Rj} - \mathbf{a}_{solpres} - \mathbf{a}_{FB} \tag{3.8}$$

$B^j \mathbf{u}^j$ only contains the nonlinear terms that differentiate the simulation from the reference orbit. The power spectral density of $B^j \mathbf{u}^j$ then gives the values for the frequencies of these nonlinear terms. Here $\mathbf{a}_{solpre}$ and $\mathbf{a}_{FB}$ are the accelerations of solar radiation pressure and third body.

3.3 – Autonomous Navigation

The control law described above requires full state feedback, i.e. the control is a linear function of the state vector. In practice, the state vector is not available, and instead all one has is an estimate of its value generated by an observer. The Kalman filter is the optimal observer, and when coupled with the LQR controller, the resulting feedback system is called a linear quadratic Gaussian, or LQG controller.

Autonomous navigation initiatives are underway at GSFC that cover spacecraft orbits that are beyond the regime in which use of GPS or the Tracking and Data Relay Satellite System (TDRSS) is feasible. GSFC is assessing the feasibility of using standard satellite attitude sensors, communication components, Doppler, crosslink systems, and GPS transceivers to provide autonomous navigation for missions that encompass libration points, gravity assist, high-Earth, and interplanetary orbits. The absolute and relative navigation solutions computed by the onboard Kalman filters can then be used in the LQG control of each spacecraft in a formation.

3.4 – Decentralized LQG Design

The model may also be cast into the form of a discrete-time distributed system, in which each satellite is assumed to locally process only its own measurements and generate its own controls, at discrete intervals. Representing the discrete time interval by the subscript i, and the sampling time by Δt, the re-cast model is:

$$\mathbf{x}_i = \Phi_i \mathbf{x}_{i-1} + \sum_{j=1}^{K} \left\{ \Lambda_i^j \mathbf{u}_i^j + \mathbf{w}_i \right\} \tag{3.9}$$

$$y_i^j = H_i^j x_i + v_i^j \tag{3.10}$$

Here, $\mathbf{x}$, Φ, and $\mathbf{w}$ are the state, state transition matrix, and process noise for all of the spacecraft in the formation, and

$$\Phi_i = \Phi(t_i + \Delta t, t_i) = e^{A \Delta t}, \quad \Lambda_i^j \mathbf{u}_i^j = \int_{t_i}^{t_i + \Delta t} \tilde{B}^j \mathbf{u}^j(t) dt \tag{3.11}$$

The matrix $\tilde{B}^j$ is the same as the matrix B^j used previously, but augmented with zeros in the appropriate locations so as to make it compatible with the state vector containing the states of all the spacecraft in the formation. The process noise is used in the Kalman filter design to accommodate model uncertainties, and y_i^j and v_i^j represent the measurements and measurement noise of the autonomous navigation system at each satellite. The covariance of the process noise is W and the covariance of the measurement noise is V^j.

A centralized solution to estimating the state for the system above would consist of the Kalman filter. The globally optimal state estimate $\hat{x}_i$ for the centralized Kalman filter depends on the

measurements from all of the spacecraft in the network. Computing $\hat{x}_i$ at each spacecraft therefore requires that all the spacecraft have all the measurement information (from all the other spacecraft), which in turn requires the transmission of large quantities of measurement data. In contrast, Speyer's approach minimizes the requirements for data transmission. Each spacecraft computes a globally optimal state estimate using only local measurements, along with information states and data vectors that compress the past measurement history into two vectors with the dimension of the state. The approach is based upon a decomposition of the state such that:

$$\hat{x}_i = \hat{x}_i^D + x_i^C \tag{3.12}$$

where $\hat{x}_i^D$ depends only on the measurements (data), and x_i^C depends only on the controls. Based on this decomposition, the local Kalman filters are modified to process only that part of the measurement that depends on the data-dependent part of the state:

$$\tilde{y}_i^j = y_i^j - H_i^j x_i^C \tag{3.13}$$

The control-dependent partition propagates according to:

$$x_i^C = \Phi_i x_{i-1}^C + \sum_{j=1}^{K} \Lambda_i^j u_{i-1}^j \tag{3.14}$$

with the initial condition: $x_1^C = x_1$. Then the data-dependent partition can be propagated and updated with all the data using the state transition matrix:

$$x_i^D = \Phi_i \hat{x}_{i-1}^D \tag{3.15}$$

with initial condition: $x_1^D = 0$. The data dependent part of the state estimate uses only locally available information at each spacecraft:

$$\hat{x}_i^{Dj} = \overline{x}_i^{Dj} + K_i^j \left[\tilde{y}_i^j - H_i^j \overline{x}_i^{Dj} \right] \tag{3.16}$$

where K_i^j is the *locally* optimal filter gain

$$K_i^j = \left(\hat{P}_i^j \right)\left(H_i^j \right)^{\mathrm{T}} \left(V_i^j \right)^{-1} \tag{3.17}$$

computed using local covariance $\hat{P}_i^j$, based only on local data. Note that $\hat{x}_i^D$ is *globally sub-optimal but locally optimal*. [Spey79] shows that the globally optimal state can be reconstructed locally, if each spacecraft transmits its information state, $\left(\hat{P}_i^j \right)^{-1} \hat{x}_i^{Dj}$, and a data vector, h_i^j, to all the other spacecraft. Then, the globally optimal state estimate is given by:

$$\hat{x}_i^D = \sum_{j=1}^{K} \left[\left(\hat{P}_i \right)\left(\hat{P}_i^j \right)^{-1} \hat{x}_i^{Dj} + h_i^j \right] \tag{3.18}$$

The data vector, which has the same dimension as the state, is maintained at each spacecraft using:

$$h_i^j = F_i h_{i-1}^j + G_i^j \overline{x}_i^{Dj} \tag{3.19}$$

where:

$$F_i \equiv \left[I - \sum_{j=1}^{K} \left\{ \hat{P}_i \left(H_i^j \right)^{\mathrm{T}} \left(V_i^j \right)^{-1} H_i^j \right\} \right] \Phi_i \tag{3.20}$$

and:

$$G_i^j = \left[F_i \hat{P}_{i-1} \left(\hat{P}_{i-1}^j \right)^{-1} \Phi_i^{-1} - \hat{P}_i \left(\overline{P}_i^j \right)^{-1} \right] \tag{3.21}$$

Note that both local and global covariance analyses must be performed to generate $\hat{P}_i$ and $\hat{P}_i^j$. In some applications (e.g. those with linear time-invariant models or those with a fixed *a priori* reference trajectory), the covariance analysis can be performed offline.

The globally optimal LQR control can be computed at each spacecraft by exchanging a set of vectors α'', given by

$$\alpha'' = \left(\Lambda^j \right)^{\mathrm{T}} S_{i-1} \Phi \left[\left(\hat{P}_i \right)\left(\hat{P}_i^j \right)^{-1} \hat{x}_i^{Dj} + h_i^j \right] \tag{3.22}$$

which have the same dimension as the control, among the spacecraft. Each spacecraft transmits α'' to and receives α'' from the other spacecraft, then computes the control according to

$$\mathbf{u}' = -\left[R' + (\mathbf{x}')^T S_{,\mathbf{x}\mathbf{x}} \mathbf{x}'\right]\left\{(\mathbf{x}')^T S_{,\mathbf{x}} \Phi \mathbf{x}' + \sum_{i=1}^{\kappa} \alpha''\right\} \qquad (3.23)$$

Note that the information exchange need only occur when a maneuver is to be performed. If the FQ algorithm (described below) is used, then instead the vectors $\mathbf{h}'$, which have the dimension of the state vector, must be exchanged at every measurement update epoch, and the globally optimal state is reconstructed.

3.5 – GSFC Controller Application

The GSFC Formation Flying Algorithm, FQ [Folt98] is adaptable to generic formation flying problems and permits full closed-loop three axis orbital maneuver autonomy onboard any spacecraft. This algorithm, to be demonstrated on the Earth Observer-1 formation flying mission, solves the position maintenance problem by combining the boundary value problem, initial and target states, and Battin's 'C*' matrix formulation to construct a state transition matrix. In this example, the goal of the algorithm is for a spacecraft to perform maneuvers which cause it to move along a specific transfer orbit. The transfer orbit is established by determining a path which will carry the spacecraft from some initial state, $(\mathbf{r}_0, \mathbf{v}_0)$, at a given time, t_0, to a target state, $(\mathbf{r}_t, \mathbf{v}_t)$, at a later time, t_t. The target state found will place the spacecraft in a location relative to the control spacecraft so as to maintain the desired formation. Back propagating the target state to find the initial state the spacecraft would need at time t_0 to achieve the target state at time t_t without executing a maneuver gives rise to the desired state, $(\mathbf{r}_d, \mathbf{v}_d)$ at time t_0. The initial state can now be differenced from the desired state to find:

$$\begin{pmatrix} \delta\mathbf{r} \\ \delta\mathbf{v} \end{pmatrix} = \begin{pmatrix} \mathbf{r}_0 - \mathbf{r}_d \\ \mathbf{v}_0 - \mathbf{v}_d \end{pmatrix} \qquad (3.24)$$

The original application of the FQ algorithm used a state transition matrix calculated using universal variables and the F&G series in a two–body formulation. For the application here, we derive the state transition matrix using the matrix exponential as shown above. That state transition matrix is then partitioned as follows:

$$\Phi(t_0,t_1) \equiv \left[\begin{array}{c} \Phi_1(t_0,t_1), \Phi_2(t_0,t_1) \\ \hline \Phi_3(t_0,t_1), \Phi_4(t_0,t_1) \end{array}\right] = \left[\begin{array}{c} \tilde{R}^*(t_0), R^*(t_0) \\ \hline \tilde{V}^*(t_0), V^*(t_0) \end{array}\right] = \left[\begin{array}{c} V^T(t_1), -R^T(t_1) \\ \hline -\tilde{V}^T(t_1), V^T(t_1) \end{array}\right] = \Phi(t_1,t_0)^{-1} \qquad (3.25)$$

Where the starred quantities are based upon the position/velocity partitions of $\Phi(t_0,t_1)$, and unstarred quantities are based on a $\Phi(t_1,t_0)$, which Battin calls the guidance matrix and navigation matrix respectively. If a reversible Keplerian path is assumed between the two states, one should expect the forward projection of the state from t_0 to t_1 to be related to the backward projection of the state from t_1 to t_0. From these sub-matrices, a C* matrix is computed as follows:

$$C^*(t_0) = V^*(t_0)\left[R^*(t_0)\right]^{-1} \qquad (3.26)$$

The expression for the impulsive maneuver applied herein follows immediately:

$$\Delta V = \left[C^*(t_0)\right]\delta\mathbf{r}_0 - \delta\mathbf{v}_0 \qquad (3.27)$$

4 – RESULTS AND DISCUSSION

The centralized and decentralized LQ controllers were used with the simulation described above, for a two spacecraft formation. The results are shown in Figures 3 and 4. Figure 3 depicts the control effort in terms of velocity increments, ΔV. The ΔVs displayed are for both spacecraft which follow their own reference orbit so that formation flying requirement can be met. While both spacecraft initially started on their respective orbits, the formation requirement is to maintain a separation of 1km. The initial offset between the spacecraft is also approximately 1km. The peak shown during the first 200 days is a result of the Q and R matrices chosen for the LQR/LQG while the periodic oscillations indicate that there are non-linear components and perturbations not accounted for in the disturbance accommodation. Continuous ΔVs expended during the first 200 days are less than 1mm/s in each velocity component.
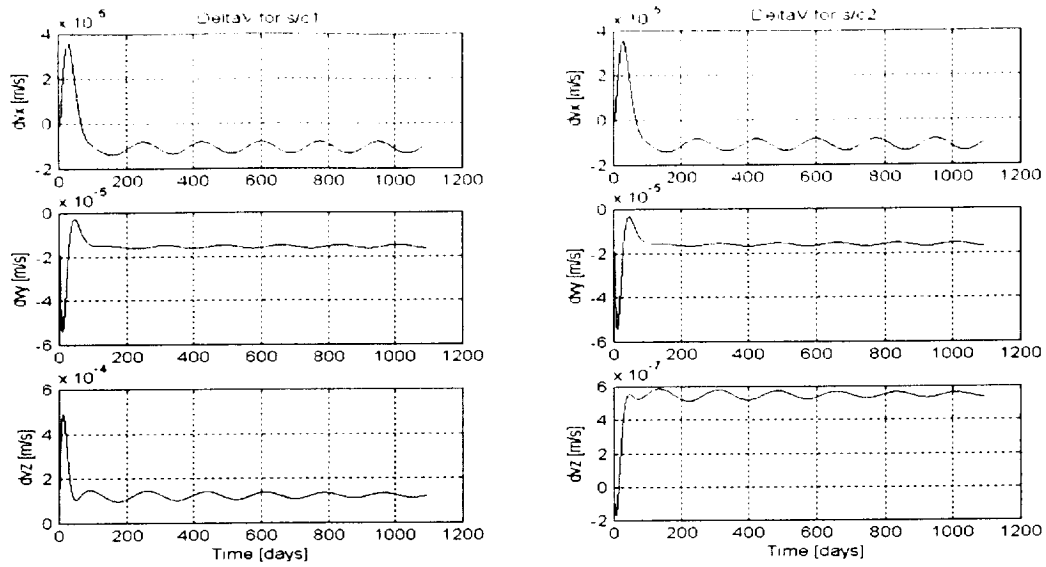
Fig. 3: ΔV to maintain libration orbit around Sun-Earth L1 without perturbations

Figure 4 shows the motion of spacecraft 2 relative to spacecraft 1, in a local coordinate system centered on spacecraft 1. This coordinate system is similar to a radial, in-track, cross-track system, in that it has its x-axis along the ray connecting $L1$ to spacecraft 1, its z-axis along the direction given by the cross-product of the x-axis and the velocity of spacecraft 1 relative to $L1$, and the y-axis completing the triad. One can see that the control requirement of 1km was maintained during the five-year simulation.
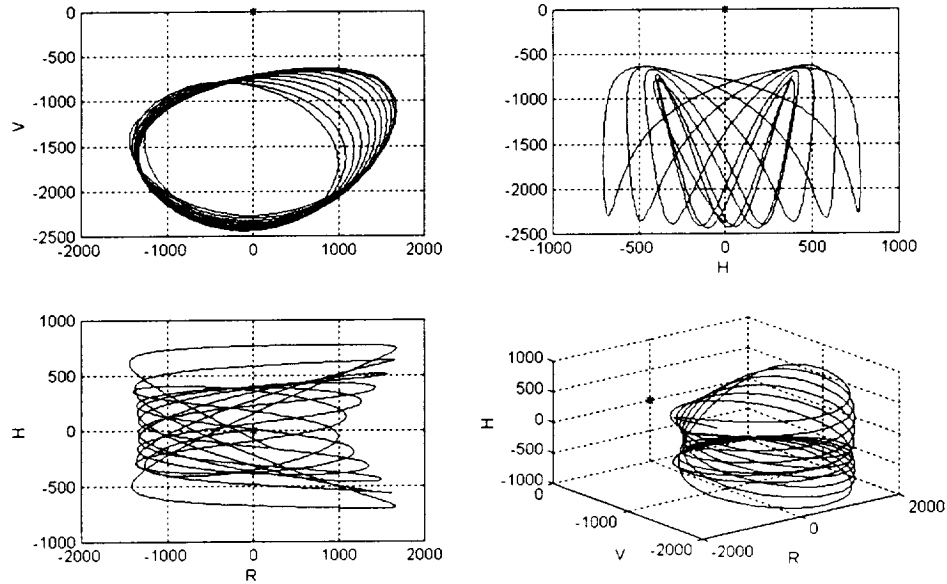

Fig. 4: Motion of spacecraft-2 with respect to spacecraft-1, in local coordinates

To demonstrate the versatility of the GSFC formation flying algorithm, control of libration orbits using initial states generated by the above state space model is investigated. The orbit selected is shown in Figure 2. The input to the GSFC algorithm is the partitioned segments of Φ, the initial, target, and desired states as described in section 3, and the time from the maneuver to the encounter of the target state. The initial state is displaced in the L1 X direction and L1 y velocity. The only maneuver constraint used is that the maneuver should occur immediately at the current position. This placed the maneuver on the X-axis of the rotating L1 coordinates. To make the targeting more complex, the target state also includes a position at various temporal displacements to show the effects on all velocity components. The position data passed into the GSFC algorithm can also be the information states or data from the decentralized method. Table 1 shows the ΔV and

information for each of the cases studied. The time interval represents the coast time from maneuver to achieving the target position with the ΔV components in the L1 X-axis and Y-axis directions. A seen in the table, the immediate x-axis ΔV required to achieve the target goals is proportional to the x state displacement while the y-axis ΔV is basically equivalent to the initial velocity displacement.

Table 1 - ΔV (m/s) to Achieve Target State in L1 Rotating Coordinate Frame

State Offset		10 Sec	60 Sec	1 Day	30 Day
X = 1km, y = 1m/s	x (m/s)	-100	-16	1.2e-2	-5.7e-4
	y (m/s)	0.9	0.9	0.9	0.9
X = 10km, y =1m/s	x (m/s)	-1000	-17	1.2e-1	-5.6e-3
	y (m/s)	1	1	1	1
X = 10km, y =10m/s	x (m/s)	-1000	-17	1.2e-1	-6.0e-3
	y (m/s)	10	10	10	10

5 – CONCLUSIONS

This work demonstrates the feasibility of using decentralized control methods to maintain a formation of satellites at the Earth-Sun co-linear libration point. Integration of decentralized control concepts within the GSFC GNCC's formation flying algorithm is a feasible technology for autonomous orbit determination and control. As science and mission requirements for virtual satellites and formation flying missions become more demanding, the benefit/cost trade of a decentralized architecture versus traditional approaches becomes promising. In order to advance this technology to the state needed to support such missions, it is essential that this innovative technology could be demonstrated on future missions. The application of this LQR, decentralized control (LQG), and the FQ maneuvering algorithms is unlimited and can be used to fully explore the NASA missions outside the Earth-Moon environment.

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